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The Art of Applying Mathematics

It is the belief of the writer of the present lines that we are witnessing the start of some extraordinary fruitful decennia in applied mathematics, or, perhaps better, the art of applying mathematics in an increasingly varied collection of phenomena. It also seems that apart from perhaps a natural swing back from an excessive preoccupation with pure mathematics this trend is mainly driven by synergetic effects, that is by the use of ideas and results of one (pure or applied) field in another to the mutual benefit of both, and by the use of intuitions belonging to one set of interpretations of a given class of models in a totally different setting in perhaps a totally different science.

In the lines above and in the statement of scope and interest of this journal, applied mathematics is to be understood in a broad sense: it includes applications of one part of mathematics into another, it includes experimental mathematics in the sense of simulations to find out things we do not yet understand and experimental mathematics in the sense of constructing models which imitate a certain puzzling set of phenomena, even when they are probably not true models in a more narrow and traditional meaning of the word; it also includes discussions of open mathematical problems generated by a (potential) application. And it most emphatically includes mathematics motivated by certain applications, also when we are not at the moment in a position to 'solve' the resulting mathematical structures.

As I remarked above, a main impetus of the current revival in applied mathematics comes from the use of ideas, concepts, results and intuitions from one area in another. Another solid bit of impetus comes from that recent phenomenon: experimental mathematics, also known as computer modeling and simulation. To flourish these interactions require communication, the finding out about the patterns of thought current in a field different from one's own; a process which tends to be difficult and which has been described as painful. It is indeed, as has been repeatedly stressed, nontrivial for, say, a medical researcher and a mathematician to achieve sufficient understanding of each other's disciplines and modes of thought in order to be able to start a truly joint project with reasonable expectations of success. It seems to me that the mathematician in such a potential partnership would be much better prepared if he had studied some survey papers written by colleagues who have been through the mill, that is who did achieve the necessary understanding of (the relevant part of) the other discipline, which are written at a level which is also accessible to mature (applied or pure) scientists of different persuasions and which are discursive enough to convey also some of the underlying intuitions of the field they deal with. It is to communication and understanding of this type that this journal aims to contribute.

Let me give a few examples of such areas of interaction (actual and potential) as I had in mind when writing the previous three paragraphs. There is, for instance, that richly promising but as yet mathematically diffusely structured area of (the mathematics of) dissipative systems; there is a flourishing area of electrical engineering: mathematical network and systems theory, in which ideas and results from algebraic and differential geometry and differential topology are deeply and fruitfully applied; there is what has been called the soliton revolution (taken to include monopoles and instantons) which has, among other things, enormously enlarged the number of models in mathematical physics which can be exactly solved and in which connections and bundles and theta functions and pseudo-differential operators play such a significant role. It is to me stimulating and fascinating to observe how ideas and concepts from combinatorics on the one hand and probability on the other are penetrating into parts of mathematics and the sciences which were, until recently, thought to be completely unrelated to these specialisms. (To be more precise as regards combinatorics, I have in mind here the interactions between partial order and such properties as Cohen–Macaulyness of commutative rings and partial order in such fields as combinatorial optimization and parts of computer science. As regards probability, I am referring, e.g., to the use of probabilistic ideas in Banach space theory and of its use to prove smoothness of solutions of partial differential equations (Malliavin Calculas) as well as the by now already almost classical interrelations between probability and classical analysis). Consider also the interaction area where algebraic and topological K-theory, index theory, C^*-algebras, foliations and physics come together. There is much more, e.g., the use of ideas from algebraic topology to discuss and analyze defects in ordered media; there is, passing from the actual to the potential, quite an array of techniques and ideas which try to exploit the occurrence of lots of cancellations in random or (almost) periodic phenomena (averaging, stationary phase, semi-classical expansions in general, aggregation techniques in economics, scattering from random media, coarse graining in thermodynamics) and it seems to me that here is a group of concepts which will repay interspecialistic examination. It has been realized that there are fruitful similarities between large ecological systems and large electrical networks and perhaps the disruptive oscillations which may occur in a power network upon the failure of one constituent have their counterpart in ecology. I also find it remarkably pleasing to read about the use of ideas from fluid dynamics to describe plant growth. In addition to all this, and as important, there has been a gradual shift in the general mathematical framework we tend to apply: it does not suffice to analyse, describe and understand a particular model or structure, one needs in addition to know its deformation, perturbation and stability properties. This is still mainly local and several application areas nowadays involve truly global phenomena which locally do not exist. Fortunately 20th century mathematics has also developed efficient techniques for linking global and local aspects.

The examples briefly alluded to are, of course, extraordinarily incomplete but they do indicate what is going on, and illustrate what is in my view only the beginning of a remarkable flowering of the art of applying mathematics.

As I remarked above, acquiring understanding in a different specialism, even to the point of being able to read the professional literature, is as a rule far from easy. Notations change, names of theorems change and so do definitions and, more importantly, there are changes in emphasis, especially as to at what points one has to be very careful and precise and where one can get away with fairly casual handwaving. There is also a natural tendency in every (applied) mathematician to rely on only the techniques one knows and trusts with as a natural result a gradually diminishing, rather than a growing bag of tools. In addition, conditioned by environment and training, one tends towards only those applications which are traditional (in that particular environment) and which have been seen to yield a sufficient measure of appreciation. Given the trends sketched above, both tendencies are, in the long run, probably counter productive.

It is perfectly stimulating to see the various specialisms and subspecialisms in mathematics and the areas of inquiry to which it is applied get intertwined, interrelated and crosscorrelated to the point where the use of a linear classification scheme becomes almost useless. But then one wants also to have the opportunity to follow all that and to participate. That requires lots of survey and state of the art papers of a reasonably discursive kind accessible to nonsuperspecialists. And that is the kind of paper this journal will try to provide.

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